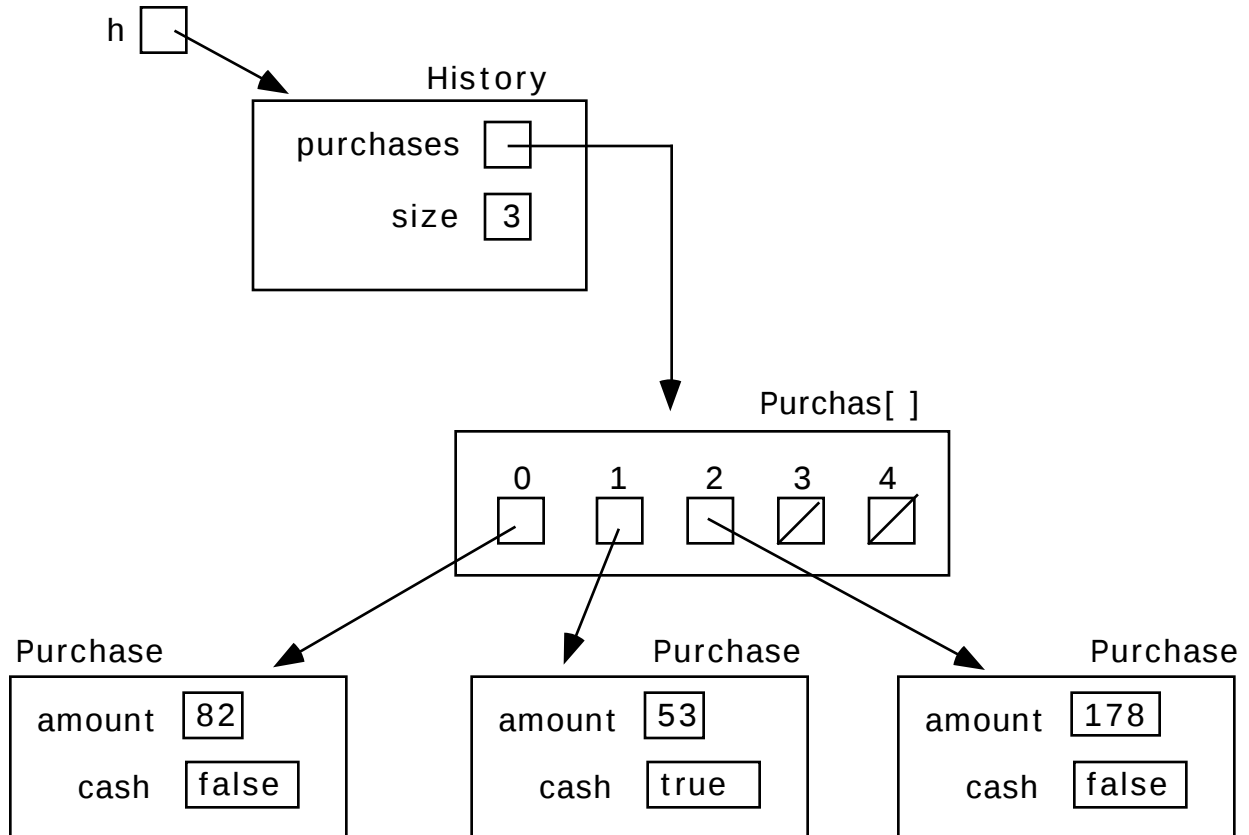


FINAL EXAM REVIEW SOLUTIONS

This final draft contains solutions to all problems.

Problem 1: Sales Statistics *(Data Abstraction, Arrays, Lists, Objects, Object Diagrams)*

Part a.



Part b. Finishing the instance methods of the History class:

```

public class History {

    // Instance Variables:
    private Purchase [ ] purchases;
    private int size;

    // Constructor Method:
    public History (int maxEntries) {
        purchases = new Purchase[maxEntries];
        size = 0;
    }

    // Instance Methods:
    public void add (int amount, boolean cash) {
        if (size < purchases.length) {
            purchases[size] = new Purchase(amount, cash);
            size = size + 1;
        }
    }
}
    
```

```

public int size(){
    return size;
}

// min() and max() are instances of the general array accumulation idiom
public int min(boolean cash){
    int minVal = Integer.MAX_VALUE; // Positive infinity is identity of min
    for (int i = 0; i < size; i++){
        if (purchases[i].getCash() == cash){
            minVal = Math.min(minVal, purchases[i].getAmount());
        }
    }
    return minVal;
}

public int max(boolean cash){
    int maxVal = Integer.MIN_VALUE; // Negative infinity is identity of max
    for (int i = 0; i < size; i++){
        if (purchases[i].getCash() == cash){
            maxVal = Math.max(maxVal, purchases[i].getAmount());
        }
    }
    return maxVal;
}

public int average(boolean cash){
    int sum = 0;
    int count = 0;
    for (int i = 0; i < size; i++){
        if (purchases[i].getCash() == cash){
            sum = sum + purchases[i].getAmount();
            count = count + 1;
        }
    }
    if (count == 0){
        return 0;
    } else {
        return sum/count;
    }
}

public int number (boolean cash){
    int count = 0;
    for (int i = 0; i < size; i++) {
        if (purchases[i].getCash() == cash){
            count = count + 1;
        }
    }
    return count;
}

public int percentage (boolean cash){
    if (size > 0){
        return (number(cash) * 100)/size;
    } else {
        return 0;
    }
}
}

```

Part c. You cannot write the constructor method as either of:

```
public Purchase (int amount, boolean cash) {
    amount = amount;
    cash = cash;
}

public Purchase (int a, boolean c) {
    int amount = a;
    int cash = c;
}
```

In both cases, the variables `amount` and `cash` refer to *local* variables in the execution frame, not the instance variables in the instance. If there is a local variable with the same name as the instance variable, the instance variable must be referred to as **this**.amount.

Part d.

1) Implement the `Purchase` class with the instance variable as an array of two integers:

```
public class Purchase {

    // Instance Variable
    private int [] sale;

    // Constructor Method
    public Purchase (int i, boolean b) {
        sale = new int [2];
        sale[0] = i;
        sale[1] = intToBool(b);
    }

    // Instance Methods
    public int getAmount () {return sale[0];}

    public void setAmount (int newAmount) {sale[0] = newAmount;}

    public boolean getCash () {return (sale[1] == 1);}

    public void setCash (boolean newCash) {
        sale[1] = intToBool(newCash);
    }

    // Useful helper method
    private bool boolToInt (boolean b) {
        if (b) {
            return 1;
        } else {
            return 0;
        }
    }
}
```

2) Implement Purchase class with an integer list:

```
public class Purchase {  
    // Instance Variable  
    private IntList sale;  
  
    // Constructor Method  
    public Purchase (int i, boolean b) {  
        sale = prepend(i, prepend(intToBool(b), empty()));  
    }  
  
    // Instance Methods  
    public int getAmount () {  
        return head(sale);  
    }  
  
    public void setAmount (int newAmount) {  
        sale = prepend(newAmount, tail(sale));  
    }  
  
    public boolean getCash () {  
        return (head(tail(sale)) == 1);  
    }  
  
    public void setCash (boolean newCash) {  
        sale = prepend(head(sale), prepend(intToBool(newCash), empty()));  
    }  
  
    // Useful helper method  
    private bool boolToInt (boolean b) {  
        if (b){  
            return 1;  
        } else {  
            return 0;  
        }  
    }  
}
```

3) Implement Purchase class as a single positive or negative integer:

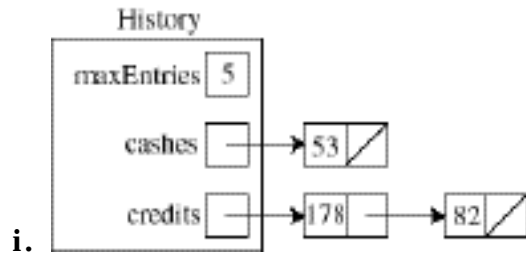
```
public class Purchase {  
    // Instance Variable  
    private int sale;  
  
    // Constructor Method  
    public Purchase (int i, boolean b) {  
        sale = (sign(b)) * i  
    }  
  
    // Instance Methods  
    public int getAmount () {  
        return Math.abs(sale);  
    }  
  
    public void setAmount (int newAmount) {  
        sale = sign(sale) * Math.abs(newAmount);  
    }  
  
    public boolean getCash () {  
        return (sale > 0);  
    }  
  
    public void setCash (boolean newCash) {  
        sale = sign(newCash) * Math.Abs(sale);  
    }  
  
    // Useful helper method  
    public int sign (boolean b) {  
        if (b) {  
            return 1;  
        } else {  
            return -1;  
        }  
    }  
}
```

4) Implement Purchase class as a single integer where (i) amount is one half of integer (via integer division) and (ii) even is cash, odd is credit.

```
public class Purchase {  
    // Instance Variable  
    private int sale;  
  
    // Constructor Method  
    public Purchase (int i, boolean b) {  
        sale = 2*i + intToBool(b);  
    }  
  
    // Instance Methods  
    public int getAmount () {  
        return sale/2; // integer division  
    }  
  
    public void setAmount (int newAmount) {  
        sale = (newAmount*2) + (sale%2);  
    }  
  
    public boolean getCash () {  
        return (sale%2) == 1;  
    }  
  
    public void setCash (boolean newCash) {  
        sale = 2*(sale/2) + intToBool(newCash);  
    }  
  
    // Useful helper method  
    private bool boolToInt (boolean b) {  
        if (b){  
            return 1;  
        } else {  
            return 0;  
        }  
    }  
}
```

Part e. Silly Bud! Making the instance variables of the Purchase class public would tie the hands of the implementer, making it more difficult to change the representation of Purchase.

Part f. Implement history with 3 instance variables: maxEntries, and two IntLists:



ii. Implementation

```

public class History {

    // Instance Variables:
    private int maxEntries;
    private IntList cashes;
    private IntList credits;

    // Constructor Method:
    public History (int maxEntries) {
        this.maxEntries = maxEntries;
        cashes = empty();
        credits = empty();
    }

    // Instance Methods:
    public void add (int amount, boolean cash) {
        if (length(cashes) + length(credits) < maxEntries) {
            if (cash){
                cashes = prepend(amount, cashes);
            } else {
                credits = prepend(amount, credits);
            }
        }
    }

    public int min (boolean cash){
        if (cash) {
            return minOfList(cashes);
        } else {
            return minOfList(credits);
        }
    }

    public int minOfList (IntList L){
        if(isEmpty(L)){
            return Integer.MAX_VALUE;
        } else {
            return Math.min(head(L), minOfList(tail(L)));
        }
    }

    public int size () {return length(cashes) + length(credits);}

    public int max (boolean cash) { //similar to min--left as exercise.}

```

```

public int average (boolean cash){
    if (cash) {
        if (isEmpty(cashes) {
            return 0;
        } else {
            return sum(cashes)/length(cashes);
        }
    } else {
        if (isEmpty(credits) {
            return 0;
        } else {
            return sum(credits)/length(cashes);
        }
    }
}

private int sum (IntList L) {
    if (isEmpty(L)) {
        return 0;
    } else {
        return head(L) + sum(tail(L));
    }
}

private int length (IntList L) {
    if (isEmpty(L)) {
        return 0;
    } else {
        return 1 + length(tail(L));
    }
}

public int number (boolean cash){
    if (cash) {
        return length(cashes);
    } else {
        return length(credits);
    }
}

public int percentage (boolean cash){
    if (size() != 0){
        if (cashes){
            return length(cashes) * 100/size();
        } else {
            return length(credits) * 100/size();
        }
    } else {
        return 0;
    }
}
}

```

Part g. Left as exercise for the student.

Problem 2: Squares (*Recursion, Iteration, TurtleWorld, BuggleWorld, PictureWorld, Java Graphics*)**Part a. Turtle World**

```
public void squares(int n, int len) {
    if (n > 0) {
        drawSquare(len);
        fd(len);
        squares(n - 1, len/2);
        bd(len);
    }
}

public void drawSquare(int length) {
    for (int i = 0; i < 4; i++) {
        fd(length);
        lt(90);
    }
}
```

Part b. Buggle World

```
public void squares(int n, int len) {
    if (n > 0) {
        bagelRect(len, len);
        forward(len);
        squares(n - 1, len/2);
        backward(len);
    }
}

// Draw a rectangle of bagels with the given width and height.
// The state of the buggle is invariant under this method.
public void bagelRect(int width, int height) {
    if (height > 0) {
        bagelLine(width);
        left();
        forward(1);
        right();
        bagelRect(width, height-1);
        left();
        backward(1);
        right();
    }
}

// Drop a line of width bagels, leaving the state of the buggle invariant
public void bagelLine(int width) {
    if (width > 0) {
        dropBagel();
        forward();
        bagelLine(width-1);
        backward();
    }
}
```

Part c. Picture World

```
public Picture squares(int n, Color c1, Color c2) {
    if (n <= 0) {
        return empty();
    }else {
        return fourPics(empty(), empty(), patch(c1),
                        squares(n-1, c2, c1);
    }
}
```

Part d. Java Graphics

```
public void squares (Graphics g, int n, int len, Color c1, Color c2) {
    int x = 0;
    int y = 0;
    while (n > 0) {
        g.setColor(c1);
        g.drawRect(x,y,len,len);

        // Update state variables of iteration
        n = n - 1;
        x = x + len;
        y = y + len/2;
        len = len/2;
        // swap colors
        Color temp = c1;
        c1 = c2;
        c2 = temp;
    }
}
```

Problem 3: Greatest Common Divisor (Iteration)

Part a. Here is the iteration table for the iterative calculation of `GCDTail(95, 60)`.

A	B
95	60
60	35
35	25
25	10
10	5
5	0

Part b.

```
public static int GCDWhile (int A, int B) {
    while (B != 0) {
        // Need a temporary variable to perform updates!
        // Could make a temp for either A or B; here we choose A.
        int oldA = A;
        A = B;
        B = oldA % B;
    }
    return A;
}
```

Part c. It is *always* possible to express a **while** loop as a **for** loop, but the result may not be particularly natural. Indeed, in this case it would not be natural to re-express Wyla's GCDTail program as a **for** loop. For loops best “fit” cases where there is a state variable controlling the number of times through the loop that is independent of other state variables. In the case of GCD, the state variable B controls the number of times through the loop, suggesting the following skeleton:

```
public static int GCDFor (int A, int B) {
    for (; B != 0; update) { // No initialization of B necessary
        statements
    }
    return A;
}
```

But what should replace *update* and *statements*? One approach is to leave *update* blank:

```
public static int GCDFor (int A, int B) {
    for (; B != 0;) { // No initialization of B necessary
        // Need a temporary variable to perform updates!
        int oldA = A;
        A = B;
        B = oldA % B;
    }
    return A;
}
```

Indeed, **for** (; test;) {statements} is always equivalent to **while** (test) {statements}. But a **for** loop that is missing both its initialization and update statements is not very compelling!

If we put $B = A \% B$ in the *update* position, then the A in $A \% B$ (which is executed after updates) is the value of A after the assignment, when we want the value of A before the assignment. We need a temporary variable to circumvent this problem, as shown below:

```
public static int GCDFor (int A, int B) {
    int oldA;
    for (; B != 0; B = oldA % B) { // No initialization of B necessary
        oldA = A;
        A = B;
    }
    return A;
}
```

This method is less straightforward than the **while** loop, which is clearer and therefore preferred.

Problem 4: Array Reversal (Arrays, Iteration)**Part a.**

```
public static int [] reverseCopy (int [] a) {
    int len = a.length;
    int [] result = new int[len];
    for (int i = 0; i < len; i++) {
        result[i] = a[(len-1)-i]; // (len-1) is last slot
    }
    return result;
}
```

Part b.

```
public static void reverseInPlace (int [] a) {
    int len = a.length;
    // Swap first (len/2) slots with last (len/2) slots
    // If len is odd, middle slot stays unchanged
    for (int i = 0; i < len/2; i++) {
        // Swap contents of slot i and (len-1)-i
        int old_a_sub_i = a[i];
        a[i] = a[(len-1)-i];
        a[(len-1)-i] = old_a_sub_i;
    }
}
```

Problem 5: Inversions (Arrays, Iteration, Lists)**Part a.**

```
// Returns the number of inversions in a:
public static int countInversions (int [] a) {
    int invs = 0; // Counts number of inversions
    for (int i = 0; i < a.length; i++) {
        for (int j = i+1; j < a.length; j++) {
            if a[i] > a[j] {
                invs = invs + 1;
            }
        }
    }
    return invs;
}
```

Part b. Suppose that PL. is the prefix used for PointList operations:

```
// Returns the inversions in a as a list of points
public static PointList listInversions (int [] a) {
    PointList points = PL.empty(); // Collects inversions as list of points
    for (int i = 0; i < a.length; i++) {
        for (int j = i+1; j < a.length; j++) {
            if a[i] > a[j] {
                points = PL.prepend(new Point(i,j), points);
            }
        }
    }
    return points;
}
```

Problem 6: Inheritance (*Inheritance*)

Statement	Printout on Java Console
System.out.println((new A()).m1());	1
System.out.println((new B()).m1());	3
System.out.println((new C()).m1());	4
System.out.println((new D()).m1());	5
System.out.println((new E()).m1());	5

Problem 7: Converting Between Different Forms of Iteration (*Lists, Arrays, Iteration*)**Part a.**

```
// Tail Recursion
public static int weightedSum (IntList L) {
    return weightedSumTail (L, 1, 0);
}

public static int weightedSumTail (IntList L, int index, int total) {
    if (isEmpty(L)) {
        return total;
    } else {
        return weightedSumTail(tail(L), index + 1, (index*head(L)) + total);
    }
}

// While loop
public static int weightedSumWhile (IntList L) {
    int index = 1;
    int total = 0;
    while (!isEmpty(L)) {
        total = total + (index*head(L));
        index = index + 1;
        L = tail(L);
    }
    return total;
}

// For loop
public static int weightedSumFor (IntList L) {
    int index = 1;
    int total = 0;
    for (; !isEmpty(L); L = tail(L);) {
        total = total + (index*head(L));
        index = index + 1;
    }
    return total;
}
```

Part b.

```
// While loop
public static boolean isMember (int n, int [] a) {
    int i = a.length - 1;
    while ((i >= 0) && (a[i] != n)) {
        i = i - 1;
    }
    return (i >= 0); // Will only be true if n is in a.
}
```

```

// For loop
public static boolean isMemberFor (int n, int [] a) {
    for (int i = a.length - 1; i>=0; i--) {
        if a[i] == n {
            return true;
        }
    }
    return false;
}

// Tail recursion
public static boolean isMemberIter (int n, int [] a) {
    return isMemberTail(n, a, a.length - 1);
}

public static boolean isMemberTail (int n, int [] a, int i) {
    if (i < 0) {
        return false;
    } else if (a[i] == n) {
        return true;
    } else {
        return isMemberTail (n, a, i-1);
    }
}

```

Part c.

```

// For loop
public static void partialSum (int [] a) {
    int sum = 0;
    for (int i = 0; i < a.length; i++) {
        sum = sum + a[i];
        a[i] = sum;
    }
}

// While loop
public static void partialSumWhile (int [] a) {
    int i = 0;
    int sum = 0;
    while (i < a.length) {
        sum = sum + a[i];
        a[i] = sum;
        i++;
    }
}

// Tail recursion
public static void partialSumIter (int [] a) {
    return partialSumTail(a, 0, 0);
}

public static void partialSumTail (int [] a, int i, int sum) {
    if (i < a.length) {
        int newsum = sum + a[i];
        a[i] = newsum;
        partialSumTail(a, i+1, newsum);
    }
}

```

Part d.

```
// Tail recursion
public static void squiggle (Graphics g, int x1, int y1, int x2, int y2) {
    if ((x1 > 0) || (y1 > 0) || (x2 > 0) || (y2 > 0)) {
        g.drawLine(x1, y1, x2, y2);
        squiggle(g, x2, y2, y1/4, x1*2);
    }
}

// While loop
public static void squiggleWhile (Graphics g, int x1, int y1, int x2, int y2) {
    while ((x1 > 0) || (y1 > 0) || (x2 > 0) || (y2 > 0)) {
        g.drawLine(x1, y1, x2, y2);
        // Introduce temporaries for x1, y1 so don't lose their values!
        old_x1 = x1;
        old_y1 = y1;
        x1 = x2;
        y1 = y2;
        x2 = old_y1/4;
        y2 = old_x1*2;
    }
}

// For loop
public static void squiggleFor (Graphics g, int x1, int y1, int x2, int y2) {
    // A particularly unconvincing example of a for loop!
    for (;(x1 > 0) || (y1 > 0) || (x2 > 0) || (y2 > 0);) {
        g.drawLine(x1, y1, x2, y2);
        // Introduce temporaries for x1, y1 so don't lose their values!
        old_x1 = x1;
        old_y1 = y1;
        x1 = x2;
        y1 = y2;
        x2 = old_y1/4;
        y2 = old_x1*2;
    }
}
```

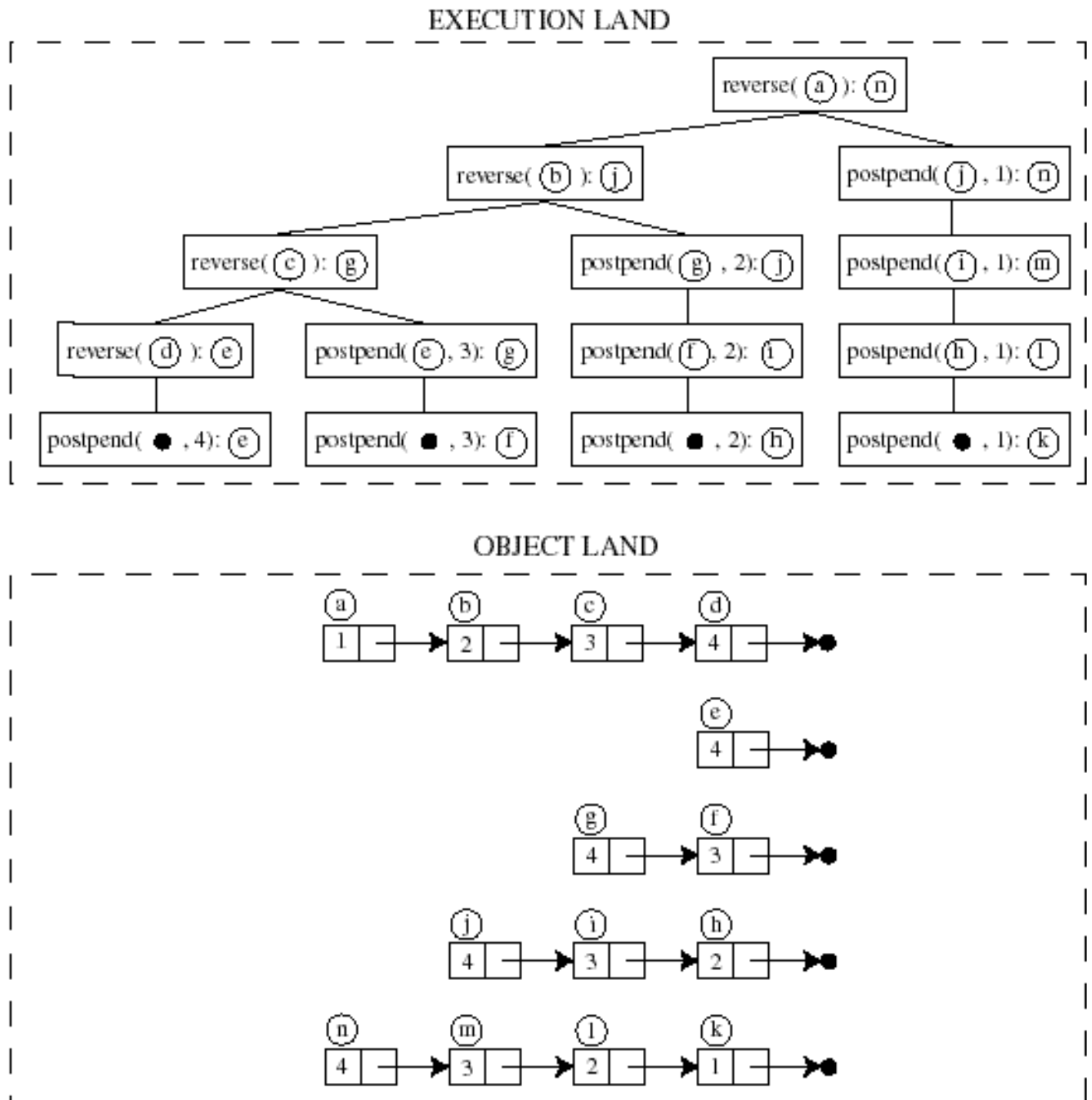
Problem 8: Converting Between Arrays and Lists (Lists, Arrays, Iteration)

```
public static int [] listToArray (IntList L) {
    int [] result = new int [IL.length(L)];
    int i = 0;
    while (! isEmpty(L)) {
        result[i] = IL.head(L);
        L = IL.tail(L);
        i = i + 1;
    }
    return result;
}

public static IntList arrayToList (int [] a) {
    IntList L = IL.empty();
    // traverse the array from back to front in order
    // to prepend elements in the correct order.
    for (int i = a.length - 1; i >= 0; i--) {
        L = prepend(a[i], L);
    }
    return L;
}
```

Problem 9: Iterative List Reversal (*Invocation Trees, Recursion, Iteration, Lists,*)

Part a.



Part b.

Tail recursive solution:

```
public static IntList reverseIter (IntList L) {
    return reverseTail(L, IL.empty());
}

public static IntList reverseTail (IntList list, IntList result) {
    if (IL.isEmpty(list)) {
        return result;
    } else {
        return reverseTail(IL.tail(list), IL.prepend(IL.head(list), result));
    }
}
```

While loop solution:

```
public static IntList reverseWhile (IntList L) {
    IntList result = IL.empty();
    while (! IL.isEmpty(L)) {
        result = IL.prepend(IL.head(L), result);
        L = IL.tail(L);
    }
    return result;
}
```

For loop solution:

```
public static IntList reverseFor (IntList L) {
    IntList result = IL.empty();
    for (; ! IL.isEmpty(L); L = IL.tail(L)) // Note empty initializer
        result = IL.prepend(IL.head(L), result);
    return result;
}
```

Problem 10: Leftist Turtles (*Tests Instance Variables, Class Declarations, Inheritance*)**Part a.**

```
public class LeftistTurtle extends Turtle {  
    private int leftCount;  
  
    public LeftistTurtle() {  
        // The super() constructor is implicitly called here.  
        leftCount = 0;  
    }  
  
    public void lt (double degrees) {  
        super.lt(degrees);  
        leftCount = leftCount + 1;  
    }  
  
    public int numberOfLefts () {  
        return leftCount;  
    }  
}
```

Part b.

The example can print 2 in the case where the `rt()` method of the `Turtle` class is implemented as follows:

```
public void rt (double degrees) {  
    left(-degrees);  
}
```

In this case, every call to `rt()` performs an implicit call on `lt()`!

Part c. In the case where the `Turtle` method `rt()` calls `lt()`, we can add a `rt()` method to `LeftistTurtle` that *subtracts* 1 from `leftCount` for each call to `rt()`. This will cancel out the 1 that is added by the implicit call to `lt()`.

```
public void rt (double degrees) {  
    super.rt(degrees);  
    leftCount = leftCount - 1;  
}
```

Suppose we don't know whether the `Turtle` `rt()` method invokes `lt()` or not – can we still handle `rt()` correctly? Yes! The following version of the `LeftistTurtle` `rt()` method is more robust than the version above because it makes no assumptions about how the `Turtle` `rt()` method works:

```
public void rt (double degrees) {  
    int oldLeftCount = leftCount;  
    super.rt(degrees); // may change leftCount  
    leftCount = oldLeftCount; // reset leftCount, in case changed by super.rt()  
}
```

Problem 11: Bank Accounts (*Data Abstraction, Lists*)**Part a.**

```
class Account {  
    private int savings;  
    private int total;  
  
    public account () {  
        this.savings = 0;  
        this.total = 0;  
    }  
  
    public int getSavings() {return this.savings;}  
    public int getChecking() {return this.total - this.savings;}  
    public int getTotal() {return this.total;}  
  
    public void depositToSavings(int amountToAdd) {  
        this.savings = this.savings + amountToAdd;  
        this.total = this.total + amountToAdd;  
    }  
  
    public void transferFromSavingsToChecking(int transferAmount) {  
        this.savings = this.savings - transferAmount;  
    }  
  
    public void withdrawFromChecking(int withdrawalAmount) {  
        this.total = this.total - withdrawalAmount;  
    }  
}
```

Part b.

```
class Account {  
    private IntList accountInfo; // List of savings, checking, total  
  
    public account () {set(0,0,0);}  
  
    // Very useful helper method  
    private set (int s, int c, int t) {  
        accountInfo = IL.prepend(s, IL.prepend(c, IL.prepend(t IL.empty())));  
    }  
  
    public int getSavings() {return IL.head(accountInfo);}  
    public int getChecking() {return IL.head(IL.tail(accountInfo));}  
    public int getTotal() {return IL.head(IL.tail(IL.tail(accountInfo)));}  
  
    public void depositToSavings(int amount) {  
        set(getSavings() + amount, getChecking(),getTotal() + amount);  
    }  
  
    public void transferFromSavingsToChecking(int amount) {  
        set(getSavings() - amount, getChecking() + ammount, getTotal());  
    }  
  
    public void withdrawFromChecking(int amount) {  
        set(getSavings(), getChecking() - amount, getTotal() - amount);  
    }  
}
```

Problem 12: End Moves (*Lists of Lists, Iteration, Recursion*)

Part a.

left	right	ans
[]	[6, 3, 7, 5]	[]
[6]	[3, 7, 5]	[[3, 7, 5, 6]]
[6, 3]	[7, 5]	[[3, 7, 5, 6], [6, 7, 5, 3]]
[6, 3, 7]	[5]	[[3, 7, 5, 6], [6, 7, 5, 3], [6, 3, 5, 7]]
[6, 3, 7, 5]	[]	[[3, 7, 5, 6], [6, 7, 5, 3], [6, 3, 5, 7], [6, 3, 7, 5]]

Part b.

```

public static endMovesWhile (IntList L) {
    IntList left = IL.empty();
    IntList right = L
    IntList ans = ILL.empty();
    while (! IL.isEmpty(right)) {
        ans = ILL.postpend(ans,
                           IL.append(left,
                                       IL.postpend(IL.tail(right),
                                                    IL.head(right))));
        left = IL.postpend(left, IL.head(right));
        right = IL.tail(right);
    }
    return ans;
}

```

Part c. Below is an alternative iteration based on the strategy suggested by the table. Note that this strategy replaces some of the calls to `postpend()` (which are expensive) with calls to `prepend()` (which are cheap).

```

public static IntListList endMovesRevIter (IntList L) {
    return endMovesRevTail (IL.empty(), IL.reverse(L), ILL.empty());
}

public static IntListList endMovesRevTail (IntList back,
                                           IntList rev,
                                           IntListList ans) {
    if (IL.isEmpty(rev)) {
        return ans;
    } else {
        return endMovesRevTail
            (IL.prepend(IL.head(rev), back),
             IL.tail(rev),
             ILL.prepend(IL.append(IL.reverse(IL.tail(rev)),
                                   IL.postpend(back,
                                                IL.head(rev))),
                        ans));
    }
}

```

Part d.

```
public static IntListList endMovesRec (IntList L) {  
    if (IL.isEmpty(L)) {  
        return ILL.empty();  
    } else {  
        return ILL.prepend(IL.postpend(IL.tail(L), IL.head(L)),  
                           ILL.mapPrepend(IL.head(L),  
                                           endMovesRec(IL.tail(L))));  
    }  
}
```